Combining Eqs. (4.7) and (4.8) with Eqs. (3.4) and the definition of Eq. (4.6), we find using $\Gamma_{ie}=\alpha^2\Gamma_{ei}$,

$$\Delta(\mathbf{k},\omega)=2i\omega^2\Gamma_{ei}/k^4. \tag{4.10}$$

Using this with Eq. (4.5), we find

$$\lim_{\omega\to 0}\frac{d\sigma(\mathbf{k},\omega)}{d\omega_b d\Omega_b}=\frac{nr_0^2}{\pi k^2}\frac{(1+\cos^2\theta)}{2}\times\left\{\frac{2}{25}\frac{(\nu_{ee}+\nu_{ii})}{k^2}+\frac{5}{6}\left(\frac{1}{\omega_{ee}}+\frac{1}{\omega_{ii}}\right)\right\}. \tag{4.11}$$

It is interesting to note that the mutual coupling term Γ_{ei} drops out at zero frequency.[13] In this expression clearly $\nu_{ii}\ll\nu_{ee}$ and $\omega_{ii}^{-1}\gg\omega_{ee}^{-1}$, but the small terms are included for the sake of symmetry. Again for a fixed ratio k/λ, $d\sigma(k,0)/d\omega d\Omega$ decreases with increasing λ. Thus, as λ increases for fixed k/λ, the acoustic resonances become higher and sharper while the zero ω part of the line becomes lower.

5. REMARKS

In this paper we have examined the effect of Coulomb collisions on the resonances in the scattering cross section. We have approximated the shape of the resonance by a Lorentzian as in Eq. (1.4). For a more detailed picture of the line shapes near and away from the resonances, the general Eq. 3.16 of I can be used. The partial cross sections $\sigma_{ss'}$ occurring in this expression can be obtained from the same conductivity calculations used above. Such detailed results on the complete scattered spectrum will be presented elsewhere.

The calculations presented here apply to weakly coupled ($k_D^3/n\ll 1$), classical ($\beta\hbar\omega\ll 1$) plasmas in the absence of an external magnetic field. If the incident radiation has a frequency high compared with the cyclotron frequency of the plasma, these results are applicable. For lower frequencies the structure[14] due to the magnetic field at multiples of the cyclotron frequency, which are predicted by the RPA, are expected to be smeared out by collisions. The effect of Coulomb collisions on this structure is an interesting problem yet to be treated.

We are grateful to Professor E. E. Salpeter for an informative discussion.

[14] E. E. Salpeter, Phys. Rev. 122, 1663 (1961).

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Resonance Line Shapes of Weak Ferromagnets of the α-Fe_2O_3 and NiF_2 Type

H. J. FINK*
Bell Telephone Laboratories, Murray Hill, New Jersey
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Based on a two sublattice model the resonance line shapes of the low- and high-frequency branch of weak ferromagnets of the α-Fe_2O_3 and NiF_2 type were calculated by solving the equations of motion with a damping term of the Landau-and-Lifshitz type. When the rf driving field is applied perpendicular to the ferromagnetic component and in the easy plane of the α-Fe_2O_3 type crystal, an enhancement factor appears in the susceptibility. The frequency linewidth is proportional to the exchange frequency and approximately independent of an applied field. The linewidths are compared with experiments and good agreement is found for $MnCO_3$. For $MnCO_3$ the damping of the high-frequency branch is by a factor of about 2.6 more effective than that of the low-frequency branch.

I. INTRODUCTION

BASED on a two sublattice model the resonance line shapes of weak ferromagnets of the α-Fe_2O_3 and NiF_2 type are calculated. Crystals of this type have two frequency branches. The low-frequency branch is an oscillation of the ferromagnetic component around its equilibrium position, and the high-frequency branch is similar to the resonance of a pure antiferromagnet. In order to calculate the linewidth we make the assumption that we may use a two sublattice model and that the damping of the resonance may be expressed by a term of the form[1] $\alpha_{lk}\mathbf{M}\times(\gamma\mathbf{M}\times\mathbf{H})/|M|$ in the equations of motion. The latter term is proportional to a torque which tends to drive the magnetic moment toward its equilibrium position. For the present considerations, α_{lk} is a phenomenological damping constant

* Present address: Atomics International, Canoga Park, California.

[1] L. D. Landau and E. M. Lifshitz, Physik. Z. Sowjetunion 8, 153 (1935).

which we shall connect with the experimentally observable quantities such as the linewidths of the low- and high-frequency branch. α_{lk} is considered a lump factor for the relaxation processes, and we shall not attempt to discuss its microscopic significance.

II. α-Fe_2O_3 TYPE WEAK FERROMAGNET

Due to the low symmetry of the crystal structure, rhombohedral crystals like α-Fe_2O_3 and $MnCO_3$ are weak ferromagnets with the ferromagnetic component perpendicular to the [111] direction.[2] When the anisotropy energy in the easy plane is neglected the free energy of the above crystals can be written as

$$E=(N/2)\langle S\rangle^2\{J(\mathbf{S}_1\cdot\mathbf{S}_2)+d\{\mathbf{k}\cdot(\mathbf{S}_1\times\mathbf{S}_2)\}+K(S_{1Z}^2+S_{2Z}^2)-\delta[\mathbf{h}\cdot(\mathbf{S}_1+\mathbf{S}_2)]\}\,, \quad (1)$$

where $\langle S\rangle$ is the average electron spin which is a function of temperature, $\mathbf{S}_1$ and $\mathbf{S}_2$ are unit vectors parallel to the sublattice magnetizations, $\mathbf{k}$ is a unit vector parallel to the [111] direction, $\mathbf{h}$ is a unit vector parallel to the applied magnetic field, J is the effective exchange energy, d is the effective anisotropic spin-spin energy,[3] K is the uniaxial anisotropy energy, and δ is equal to $g\mu_B H/S$. The real and imaginary rf susceptibilities are calculated from

$$d\mathbf{M}_i/dt=\gamma\mathbf{M}_i\times\mathbf{H}_i-\alpha_{lk}(\mathbf{M}_i/|M|)\times(\gamma\mathbf{M}_i\times\mathbf{H}_i)\,, \quad (2)$$

where $\mathbf{H}_i$ is the effective field of the ith sublattice and the latter is calculated from $\mathbf{H}_i=-\partial E/\partial\mathbf{M}_i$. Because the effective anisotropy field of the above crystals is small in the (111) plane, we consider the case for which the external field is always larger than the effective anisotropy field in the (111) plane. The ferromagnetic component is then always parallel to the component of the applied static field in the (111) plane and this direction is called the x direction. The z direction is parallel to the [111] direction and therefore the static field H is assumed to be always in the (xz) plane.

In order that the zero field resonance of the low-frequency branch does not occur at zero frequency we must introduce a small but finite anisotropy field in the (111) plane. We can do that by adding an anisotropy field H_A' approximately parallel to the sublattice magnetizations. In our calculations H_A' is assumed to be along the y axis.[4] When the rf susceptibilities are calculated with the usual assumptions of linearizations of the equations of motion, and the differential operator d/dt is replaced by $i\omega$, one obtains

$$\frac{\chi_{xx}}{\chi_0}=f'+if''\,, \quad (3)$$

$$\frac{\chi_{yy}}{\chi_0}=\frac{H_{DM}+H_x}{H^*+H_x}(g'+ig'')+\frac{H_z^2}{2H_EH_A+H_z^2}(f'+if'')\,, \quad (4)$$

$$\frac{\chi_{zz}}{\chi_0}=g'+ig''\,, \quad (5)$$

$$\frac{\chi_{xy}}{\chi_0}=\frac{\chi_{yx}^*}{\chi_0}=\left(\frac{\omega}{\gamma}\right)\frac{H_z}{2H_EH_A+H_z^2}(f''-if')\,, \quad (6)$$

$$\frac{\chi_{yz}}{\chi_0}=\frac{\chi_{zy}^*}{\chi_0}=\left(\frac{\omega}{\gamma}\right)\frac{1}{H^*+H_x}(g''-ig')\,, \quad (7)$$

$$\frac{\chi_{xz}}{\chi_0}=\frac{\chi_{zx}^*}{\chi_0}=\frac{\gamma^2H_xH_z\omega^2}{\omega_+^2\omega_-^2}\times[(f'g'-f''g'')+i(f'g''+f''g')]\,. \quad (8)$$

The expressions f', f'', g', and g'' are defined by

$$f'=\frac{(\omega_-^2-\omega^2)\omega_-^2}{(\omega_-^2-\omega^2)^2+\omega^2(\Delta\omega_-)^2}\,, \quad (9)$$

$$f''=\frac{\omega\omega_-^2(\Delta\omega_-)}{(\omega_-^2-\omega^2)^2+\omega^2(\Delta\omega_-)^2}\,, \quad (10)$$

$$g'=\frac{(\omega_+^2-\omega^2)\omega_+^2}{(\omega_+^2-\omega^2)^2+\omega^2(\Delta\omega_+)^2}\,, \quad (11)$$

$$g''=\frac{\omega\omega_+^2(\Delta\omega_+)}{(\omega_+^2-\omega^2)^2+\omega^2(\Delta\omega_+)^2}\,. \quad (12)$$

The other quantities for the $k=0$ mode are

$$\chi_0=M/H_E\,, \quad (13)$$

$$\omega_+^2=\gamma^2[2H_EH_A'+H_x(H_x+H_{DM})]\,, \quad (14)$$

$$\omega_-^2=\gamma[2H_EH_A+H_{DM}(H_{DM}+H_x)+H_z^2]\,, \quad (15)$$

$$H^*=2H_EH_A'/H_{DM}\,, \quad (16)$$

where the exchange field H_E is equal to $SJ/g\mu_B$, the field due to the anisotropic spin-spin interaction[3] H_{DM} is dH_E/J, the uniaxial anisotropy field H_A parallel to the [111] is $2KH_E/J$, H_A' is the anisotropy field in the (111) plane which is along the y direction and almost parallel to the sublattice magnetizations, and H is the applied static magnetic field. The frequency linewidth $\Delta\omega$ is then to the second order in ω_E, where $\omega_E(=\gamma H_E)$

[2] I. E. Dzialoshinskii, Zh. Eksperim. i Teor. Fiz. **32**, 1547 (1957) [English transl.: Soviet Phys.—JETP **5**, 1259 (1957)].

[3] T. Moriya, Phys. Rev. **120**, 91 (1960); Chapter on "Weak Ferromagnetism" in *Magnetism* edited by G. T. Rado and H. Suhl (to be published).

[4] When a hyperfine interaction term exists, part of H_A' is due to the nuclear polarization which becomes important at low temperatures as it increases inversely to the temperature. This will effect Eqs. (4), (7), (14), and (16). See, for example: D. Shaltiel and H. J. Fink, Suppl. J. Appl. Phys. (to be published).

is the exchange frequency:

$$\Delta\omega_+=\omega_E\left\{2\alpha_+ +\alpha_+\left[\frac{1}{2}\left(\frac{\omega_-}{\omega_E}\right)^2-\left(\frac{H_z}{H_E}\right)^2\right]+\frac{\alpha_-}{2}\left(\frac{\omega_+}{\omega_E}\right)^2\right\}\approx 2\alpha_+\omega_E, \quad (17)$$

$$\Delta\omega_-=\omega_E\left\{2\alpha_- +\alpha_-\left[\frac{1}{2}\left(\frac{\omega_+}{\omega_E}\right)^2-\left(\frac{H_{DM}}{H_E}\right)^2\right]+\frac{\alpha_+}{2}\left(\frac{\omega_-}{\omega_E}\right)^2\right\}\approx 2\alpha_-\omega_E. \quad (18)$$

The approximate expressions for Eqs. (17) and (18) are valid because $\omega_E\gg\omega_\pm$ for $H\ll H_E$. Therefore, $\Delta\omega_+$ and $\Delta\omega_-$ are approximately constant and independent of an applied field, provided $\alpha_\pm$ is constant. α_+ refers to a phenomenological damping term in the (111) plane which is associated with the low-frequency branch, and α_- refers to a damping term perpendicular to the (111) plane which is connected with the high-frequency branch.

When no external static magnetic field is applied, an rf magnetic field parallel to the ferromagnetic component stimulates the high-frequency resonance and an rf field perpendicular to the ferromagnetic component induces the low-frequency resonance. From Eqs. (3) to (12) it can be seen that this restriction is lifted when a static magnetic field is applied in an arbitrary direction with respect to the crystal axes.

It should be noted that the expression $(H_{DM}+H_x)/(H^*+H_x)$ in Eq. (4) can be large compared to unity for $H_x\to 0$ as will be shown below. This term is an enhancement factor when $H_{DM}\gg H^*$.

The resonance frequencies ω_+ for the low-frequency branch and ω_- for the high-frequency branch have been calculated previously.[3–8] The maximum absorption as a function of frequency occurs at a frequency

$$(\omega_\pm)_{\max}\approx\omega_\pm[1+\tfrac{3}{8}(\Delta\omega_\pm/\omega_\pm)^2)], \quad (19)$$

and the total linewidth D at the half-power points is

$$D_\pm\approx\Delta\omega_\pm[1-\tfrac{1}{4}(\Delta\omega_\pm/\omega_\pm)]. \quad (20)$$

If one makes the assumption that $\Delta\omega_\pm\ll\omega_\pm$ the resonance line shapes become Lorentzian and the magnetic field linewidths of the low- and high-frequency branch, respectively, reduce to

$$\Delta H_+\approx\frac{4\alpha_+H_E(\omega_+/\gamma)}{H_{DM}\sin\theta+2H\sin^2\theta}, \quad (21)$$

$$\Delta H_-\approx\frac{4\alpha_-H_E(\omega_-/\gamma)}{H_{DM}\sin\theta+2H\cos^2\theta}, \quad (22)$$

where θ is the angle between the [111] direction and the magnetic field. In obtaining Eq. (21) and (22) it was assumed that $dH/d\omega$ can be replaced by $\Delta H/\Delta\omega$. The magnetic field linewidth for $MnCO_3$ has been measured[9,10] and we shall compare theory and experiment below.

III. NiF_2 TYPE WEAK FERROMAGNETS

NiF_2 has a rutile-type crystal structure. Below 73°K it is a weak ferromagnet. The weak ferromagnetic component arises from the single ion anisotropy[11] and it points along the ⟨100⟩ direction[12] when no external magnetic field is applied. The free energy consistent with the crystal symmetry to terms of second order can be written as[13]

$$E=(N/2)\langle S\rangle^2[J(\mathbf{S}_1\cdot\mathbf{S}_2)+K(S_{1z}^2+S_{2z}^2)+d(S_{1a}S_{1b}-S_{2a}S_{2b})-\delta\mathbf{h}\cdot(\mathbf{S}_1+\mathbf{S}_2)], \quad (23)$$

provided the exchange interaction is isotropic. K is the uniaxial anisotropy energy associated with the [001] direction (z direction) and d is the anisotropy energy in the (001) plane. [100] is the a direction and [010] is the b direction. For simplicity we assume that the static field is large enough to order the domains and that it is applied in the (010) plane, and that the ferromagnetic component is parallel to the a direction. Then one obtains for NiF_2 similar results as above by replacing in Eqs. (3) to (13) x by a, y by b, H_{DM} by H_M, and H^* by $4H_M$. The resonance branches for the $k=0$ mode are[11]

$$\omega_+^2=\gamma^2(H_a+H_M)(4H_M+H_a), \quad (24)$$

$$\omega_-^2=\gamma^2[2H_EH_A+H_M(H_M+H_a)+H_z^2], \quad (25)$$

where $H_M=H_Ed/J$ and H_a the applied field along the a direction. The parameters K and d in Eq. (23) correspond to the crystal field parameters D and $2E$, respectively, in Moriya's spin Hamiltonian.[11] The linewidth and the maximum of the absorption are again given by Eq. (17) to (20) with the above appropriate substitutions. Here α_+ refers to a damping term in the (001) plane which is again associated with the low-frequency branch, and α_- refers to a damping term perpendicular to the (001) plane which is again con-

[5] A. S. Borovik-Ramonov, Zh. Eksperim. i Teor. Fiz. **36**, 766 (1959) [English transl.: Soviet Phys.—JETP **9**, 539 (1959)].
[6] P. Pincus, Phys. Rev. Letters **5**, 13 (1960).
[7] E. A. Turov and N. G. Gusseinov, Zh. Eksperim. i Teor. Fiz. **38**, 1326 (1960) [English transl.: Soviet Phys.—JETP **11**, 955 (1960)].
[8] H. J. Fink, Phys. Rev. **130**, 177 (1963).
[9] H. J. Fink and D. Shaltiel, Phys. Rev. **130**, 627 (1963).
[10] M. Date, J. Phys. Soc. Japan **15**, 2251 (1960).
[11] T. Moriya, Phys. Rev. **117**, 635 (1960).
[12] L. M. Matarrese and J. M. Stout, Phys. Rev. **94**, 1792 (1954).
[13] I. E. Dzialoshinskii, Zh. Eksperim. i Teor. Fiz. **33**, 1454 (1957) [English transl.: Soviet Phys.—JETP **6**, 1120 (1958)].

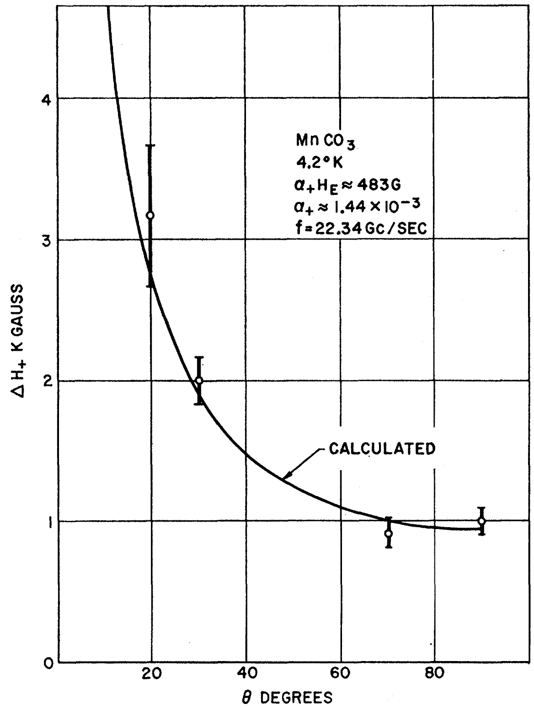

FIG. 1. The angular dependence of the resonance linewidth of the low-frequency branch of $MnCO_3$. The experimental points are by Date[10] and the solid line is calculated from Eq. (21). When $\theta=0°$, the static magnetic field is parallel to the [111] direction.

nected with the high-frequency branch. The high- and low-frequency branch have been discovered by Richards.[14,15] From the low-frequency branch it follows that $H_M=14.3$ kG (d is equivalent to 1.55 wave numbers). The exchange energy can be calculated from $J=\frac{3}{4}k(T_N+\theta_p)$, where $T_N=73.2°K$[16] and $\theta_p\approx 115°K$.[17] With $g=2.33$[18] it follows that $H_E=904$ kG and therefore one obtains from the high-frequency branch $H_A=40.2$ kG (K is equivalent to 2.19 wave numbers).

IV. COMPARISON OF THEORY AND EXPERIMENT

The magnetic field linewidths of $MnCO_3$ of the low-frequency[10] and high-frequency branch[9] have been measured. In Fig. 1, Eq. (21) was fitted to Date's results[19] with $\alpha_+H_E=483$ G. There is good agreement between the calculated and observed values. Because the magnetic-field linewidths of the high-frequency branch are very broad and because of the assumptions made in deriving Eq. (22), we cannot use Eq. (22) to compare the experimental results in Fig. 4 of Ref. 9. We, however, can use a graphical method to show that

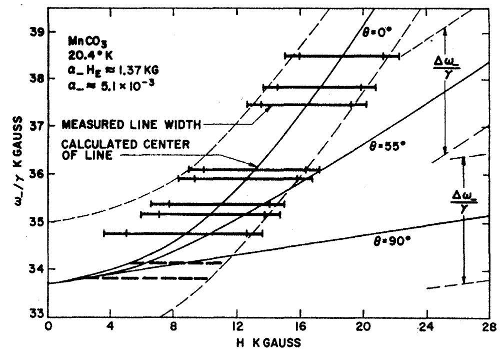

FIG. 2. Comparison of the magnetic and frequency linewidth of the high-frequency branch of $MnCO_3$ as a function of frequency and applied magnetic field. For $\theta=0°$ the static magnetic field is parallel to the [111] direction. The horizontal lines are measured magnetic-field linewidths.[9] The center of the resonance and the frequency linewidth $\Delta\omega_-/\gamma$ are also indicated for $\theta=55°$ and $90°$.[20]

$\Delta\omega$ is approximately a constant for all the fields and frequencies used in the experiment. In Fig. 2 the experimental data[9] are plotted as a function of ω_-/γ and field. The center of the lines are calculated from Eq. (15) with $2H_EH_A+H_{DM}^2=1135$ kG^2, $H_{DM}=3.62$ kG and $g=2.03$.[20] The measured linewidths are indicated as

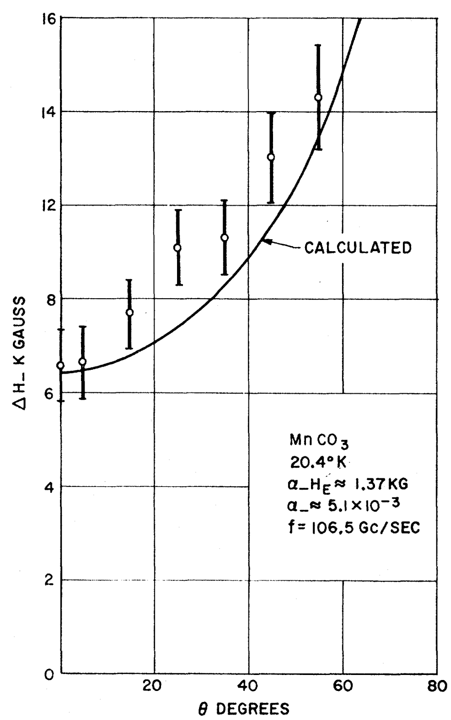

FIG. 3. Comparison between the measured[9] angular dependence of the half-linewidth of the high-frequency resonance of $MnCO_3$ and that calculated from Eq. (22). When the magnetic field is parallel to the [111] direction, the angle $\theta=0°$.

[14] P. L. Richards, Suppl. J. Appl. Phys. **34**, 1237 (1963).

[15] P. L. Richards, Suppl. J. Appl. Phys. (to be published).

[16] A. M. Cook and R. Zazenby (unpublished).

[17] H. Bizette, J. Phys. Radium **12**, 161 (1951) (for MnF_2, $\theta_p=113°K$ and for FeF_2, $\theta_p=117°K$).

[18] M. Peter and J. B. Mock, Phys. Rev. **118**, 136 (1960).

[19] In Ref. 10, Figs. 3, 4, and 5 are not consistent for an angle of 30° and 40°. The angles should be 20° and 30°. The appropriate corrections were made in Fig. 1.

[20] Here a g value of 2.03 was used. It should be considered as an effective g value. It is larger than one obtains from paramagnetic resonance for Mn. It was obtained by the best fit of the experimental results in Ref. 9 and it might be partially different due to demagnetization effects.

horizontal lines and we find that $\Delta\omega_-/\gamma$ is equivalent to 0.27 wave numbers or $\alpha_- H_E = 1.37$ kG. From Fig. 2 it is obvious why the magnetic-field lines broadened appreciably at low fields and also when the angle θ between the static magnetic field and the [111] direction was increased. The reason is that $\Delta\omega_-$ remains a constant. For $\theta = 90°$ the magnetic field line was too broad to be observed. For large magnetic fields, Eq. (22) applies approximately, and we have plotted Eq. (22) in Fig. 3 with $\alpha_- H_E = 1.37$ kG for a frequency of 106.5 Gc/sec. There is fair agreement between the calculated values and the experimental results (Fig. 5, Ref. 9). We have also measured the linewidth of the low-frequency branch for the same crystal as shown in Figs. 2 and 3. At 47.7 Gc/sec, 20.4°K, and $\theta = 90°$, we obtained[21] for $\Delta H_+ = 0.93$ kG from which it follows that $\alpha_+ H_E = 521$ G and $\alpha_-/\alpha_+ \approx 2.6$. Therefore, the damping associated with high-frequency branch is by a factor of 2.6 more effective than that associated with low-frequency branch. For a synthetic crystal of $MnCO_3$ we obtained[21] at 47.7 Gc/sec, 20.4°K, and $\theta = 90°$, a linewidth $\Delta H_+ = 0.52$ kG which shows that the natural crystals used in the above experiments were not perfect.

There are not sufficient experimental data available on the linewidth of NiF_2 to make any detailed comparison, except that $\alpha_-/\alpha_+ > 1$.[14,15]

V. FURTHER REMARKS

Crystals of the α-Fe_2O_3 structure have the ferromagnetic component in the easy plane. If an rf magnetic field h_y is applied in the easy plane perpendicular to the ferromagnetic component m_0 (with $h_x = h_z = H_z = 0$), then m_0 deflects from its equilibrium position in the (111) plane when the resonance frequency ω_+ is applied. The angle of deflection ϵ_y in the (111) plane is

$$\epsilon_y = (h_y/2\alpha_+ H_E)(H_{DM} + H_x)/(H^* + H_x) \qquad (26)$$

and out of the (111) plane:

$$\epsilon_z = h_y/2\alpha_+ H_E. \qquad (27)$$

For example for α-Fe_2O_3 at 300°K we measured[21] $\Delta H_+ \approx 200$G for a 0.018-in. sphere at 47.7 Gc/sec when the static magnetic field was perpendicular to the [111] direction. For α-Fe_2O_3 we have $H_{DM} = 22.8$ kG and $(2H_E H_A')^{1/2} = 1.45$ kG,[7,22] from which it follows that $\epsilon_y \approx 3.7°$ ($H_x \to 0$) when an rf driving field of 1 G is applied. For a crystal of $MnCO_3$ the deflection would be one order of magnitude smaller at 20.4°K. When $H_x \gg H_{DM}$, $\epsilon_y \approx \epsilon_z$ and the ferromagnetic component describes a circular path.

The ferromagnetic component will also be changed by the high-frequency resonance. For example for $h_z = h_y = H_x \to 0$ we obtain

$$m_x/m_0 = (h_x/2\alpha_- H_E)[(\omega_-/\gamma)/H_{DM}]. \qquad (28)$$

For $MnCO_3$ at 106.5 Gc/sec, 20.4°K, $\alpha_- H_E = 1.37$ kG, and $h_x = 1$ G, it follows that $m_x/m_0 \approx 3.7 \times 10^{-3}$. The ferromagnetic component extends and contracts by about 0.37% per gauss rf driving field parallel to its equilibrium length.

VI. CONCLUSIONS

The above calculations show that the resonance line shapes of both frequency branches of a weak ferromagnet are not Lorentzian. The exact functional dependence of the line shapes [Eqs. (9) to (12)] are different from those of a uniaxial antiferromagnet[23,24] but for $\Delta\omega_\pm \ll \omega_\pm$ the frequency line shapes become Lorentzian and $\Delta\omega_\pm$ becomes approximately a constant and independent of an applied magnetic field as for an antiferromagnet. For $MnCO_3$ the damping constants associated with the high- and low-frequency branch differ by a factor of 2.6 at 20.4°K for the same crystal. There is good agreement between the calculated and experimental results of the resonance linewidth of $MnCO_3$. When the rf driving field is applied in the easy plane of the α-Fe_2O_3 structure and perpendicular to the ferromagnetic component, an enhancement factor appears in the susceptibility [Eq. (4)], which for $H_x \to 0$ becomes $H_{DM}^2/2H_E H_A'$, which is large compared to unity.

In α-Fe_2O_3 and $MnCO_3$ moderate driving fields deflect, elongate, and contract the ferromagnetic component by a considerable amount. One would expect therefore that nonlinear effects play a dominant role in the above crystals when interacting with microwaves.

ACKNOWLEDGMENTS

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[21] D. Shaltiel and H. J. Fink (unpublished results).

[22] H. Kumagai, H. Abe, K. Ôno, I. Hayashi, J. Shimada, and K. Iwanaga, Phys. Rev. **99**, 1116 (1955).

[23] M. I. Kaganov and V. M. Tsukernik, Zh. Eksperim. i Teor. Fiz. **34**, 524 (1958) [English transl.: Soviet Phys.—JETP **6**, 361 (1958)]; Zh. Eksperim. i Teor. Fiz. **41**, 267 (1961) [English transl.: Soviet Phys.—JETP **14**, 192 (1962)].

[24] R. C. Ohlman and M. Tinkham, Phys. Rev. **123**, 425 (1961).